Sobol’s Method and the DICE Model

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Key Points

• Technical Focus:
  – Which (of many) exogenous DICE parameters control the uncertainties associated with climate abatement and damages costs?
  – How extreme are these costs for plausible states-of-the-world?
  – How interdependent are controlling DICE assumptions and parameters?
  – How do these dependencies evolve over time?

• What is the relevance of the results?
  – Motivation for better characterization of multivariate parameter uncertainties and modeling assumptions
  – Avoid the false assumption that isolated single factor studies are appropriate
  – Aid our understanding of the tradeoff between investing in abatement versus reacting to damages
How do we gain confidence in the models we use?

Sobol’ SA: global variance decomposition

For a simple example, with three uncertain parameters:

Total variance: \[ V(Y) = V_1 + V_2 + V_3 + V_{12} + V_{23} + V_{13} + V_{123} \]

First order sensitivity index for Parameter 1: \[ S_1 = \frac{V_1}{V} \]

Total order sensitivity index for Parameter 1: \[ S_{T1} = 1 - \frac{V_{\sim 1}}{V} = 1 - \frac{V_2 + V_3 + V_{23}}{V} \]
How can we compute the sensitivity indices efficiently?

1. Make \( N(k+2) \) samples varying parameters \( (X_1, \ldots, X_k) \) within given bounds using quasi-random sampling (Sobol’ sequences) to minimize clumps and gaps in the samples.

2. Run the model for each parameter sample.

3. Calculate the total variance: \( V(Y) \)

4. Make Monte Carlo estimates of the conditional variances: \( V[E(Y|X_i)] \) and \( V[E(Y|X_{\sim i})] \)

5. Calculate the sensitivity indices.

6. Use bootstrapping to calculate uncertainty estimates for the sensitivity indices.
Improving Convergence

Uniform Random

Sobol’ Sequence

$\frac{1}{\sqrt{N}}$  
Error Growth Rate  
$\frac{1}{N}$
DICE2007 is a globally-aggregated Integrated Assessment Model

**Resources:**
- Labor
- Capital
- Technology
- Fossil Fuel

**Optimize:**
- Consumption
- Production
- Abatement
- Policy

**Resources:**
- backstop technology
- emission controls
- carbon intensity
- participation in abatement

**Industrial emissions:**
- land use change emissions

**Carbon Cycle:**
- atmospheric C
- upper strata C
- lower strata C

**Radiative forcing:**
- atmospheric C
- Other forcing (exogenous)
- deep ocean T

**Climate Damage:**
- temperature change

**Policy:**
- climate damage

**Investment:**
- investment

**Consumption:**
- consumption
Which of these many exogenous factors are important?
We analyze sensitivities to 31 exogenous parameters

Resources:

- Labor (POP0, GPOP0, POPASYM)
- Capital (K0, DK)
- Technology (A0, GA0, DELA, PBACK, BACKRAT, GBACK, FOSSLIM)
- Fossil Fuel

Optimize:

- Investment (SAVERATE0, GAMA)
- Abatement (THETA2, PARTFRAC1, PARTFRAC2, PARTFRAC2N, DPARTFRAC)
- Policy (LIMMIU, A1, A2, A3)

- Consumption (ELASMU, PRSTP, scale1, scale2)
- Production (SIG0, GSIGMA, DSIG, DSIG2)

- Carbon Cycle
  - Industrial emissions
  - Land use change emissions
  - Radiative forcing (atmospheric C, b12, b12)
  - Other forcing (exogenous)
  - Deep ocean T
  - Climate

DICE2007
Nordhaus (2008)
We pull the DICE model out of GAMS to analyze the model rather than the solver.

- We use the DICE GAMS emission control rates to define strategies that trade off climate damages and abatement.
- We explore the sensitivity of the DICE model relative to each strategy.
- The Business-as-Usual (BAU) strategy defers abatement beyond the typical planning horizon.
We pull the DICE model out of GAMS to analyze the model rather than the solver.

- We use the DICE GAMS emission control rates to define strategies that trade off climate damages and abatement.
- We explore the sensitivity of the DICE model relative to each strategy.
- The Optimal Strategy slowly ramps up abatement.
We pull the DICE model out of GAMS to analyze the model rather than the solver.

- We use the DICE GAMS emission control rates to define strategies that trade off climate damages and abatement.
- We explore the sensitivity of the DICE model relative to each strategy.
- The doubled-CO$_2$ stabilization strategy requires earlier abatement.
We pull the DICE model out of GAMS to analyze the model rather than the solver.

- We use the DICE GAMS emission control rates to define strategies that trade off climate damages and abatement.
- We explore the sensitivity of the DICE model relative to each strategy.
- The 2°C stabilization strategy is more aggressive in implementing abatement.

We use a variance-based global sensitivity analysis method on relevant model outputs/measures. With over 8 million SOWs, we can also report probabilities of meeting stabilization targets.
A Guide to the 31 Exogenous Parameters

First Order Indices are solid circles

Total Order Indices are rings

Second Order Indices are connecting lines

Diameters of circles and rings and the width of lines are proportional to the indices.

Only indices > 1% are shown.
An Example Metric: NPV of Total Costs, BAU Policy

**Climate Sensitivity \( t_{2xco2} \)**
- First Order: 8%
- Total Order: 46%

This metric is sensitive to the assumption of a quadratic damage function in DICE.

**Climate damages exponent \( a_3 \)**
- First Order: 11%
- Total Order: 59%

**Population limit \( \text{popasym} \)**
- First Order: 3%
- Total Order: 22%

Most metrics are sensitive to this limit.

**Initial rate of change of carbon intensity \( g_{sigma} \)**
- First Order: 1%
- Total Order: 14%

**Initial rate of change of total factor productivity \( g_{a0} \)**
- First Order: 4%
- Total Order: 31%

**Savings rate \( \text{saverate0} \)**
- First Order: 6%
- Total Order: 38%

**Second order indices (lines) 1 to 8%**
Sensitivities for NPV Total Costs by Strategy

BAU

optimal

2xCO₂

2°C
Rank Order of Sensitivities of Decadal Total Costs in 100 years differ by abatement strategy.
We begin to see sensitivities to abatement parameters in 2°C strategy.
NPV Total Costs of 2°C Strategy

**Climate Sensitivity \( t2xco2 \)**
- First Order: 9%
- Total Order: 18%

**Savings rate \( saverage0 \)**
- First Order: 34%
- Total Order: 48%

This accounts for half of the first order sensitivity.

**Initial Backstop Price \( pback0 \)**
- First Order: 9%
- Total Order: 14%

**Participation \( partfrac2, partfracn \)**
- First Order: 3%
- Total Order: 7%
NPV Total Costs of 2xCO$_2$ Strategy

**Climate Sensitivity t2xco2**
- First Order: 13%
- Total Order: 38%

**Climate damages exponent a3**
- First Order: 10%
- Total Order: 34%

**Savings rate saverate0**
- First Order: 21%
- Total Order: 44%
Using the emissions control rate from the DICE 2xCO$_2$ stabilization case, 45% of the SOWs exceed the target.

This CDF describes the distribution of atmospheric concentrations in the SOWs in 100 years (approximately the maximum atmospheric concentration levels).

The median concentration is 10 ppm below the 2xCO$_2$ target.

For 97% of the SOWs, the maximum CO$_2$ concentration is less than 2.5 x pre-industrial levels.

Vertical dashed line is the value for the deterministic result.
The median decadal costs as a percent of consumption in 100 years are much greater than the deterministic result.

Only 14% of the SOWs have decadal costs less than the 3.6% value for the deterministic optimized case.
Breaking down the 2xCO₂ strategy net present value total costs into climate damages and abatement cost components

Climate Damages

Abatement Costs

Note: These net present value metrics are calculated using discount factors derived from the model output ‘return on capital’ (aka real interest rate), not from the time preference rate.
For decadal abatement costs, the sensitive parameters and the interactions change over time.
Key Points Summary

• There appears to be a strong nonlinear amplification of costs in alternatives SOWs.
• Driving uncertainties differ dramatically with mitigation strategy (aggressive abatement vs. inaction)
• The problem is non-separable. Parameter interactions are significant and change over time, both in number and in degree.
• Controlling DICE uncertainties and modeling assumptions:
  – technology efficiency (total factor productivity)
  – population growth dynamics
  – climate sensitivity
  – climate damage formulation
  – participation in abatement
• A question of insensitivity: Should climate matter more?
Questions?
Extras
NPV Total Costs: BAU and Optimal Strategy sensitivities are similar.

The total costs are dominated by climate damages in the current century. Most of the abatement costs for both of these strategies occurs beyond 2100 when they contribute less to the net present value.
NPV Total Costs Optimal Strategy: Parameter interactions are again important.

**Initial rate of change of total factor productivity**

\[ ga0 \]
- **First Order:** 4%
- **Total Order:** 23%

**Climate Sensitivity**

\[ t2xco2 \]
- **First Order:** 11%
- **Total Order:** 43%

**Climate damages exponent**

\[ a3 \]
- **First Order:** 11%
- **Total Order:** 45%

**Population limit**

\[ popasym \]
- **First Order:** 4%
- **Total Order:** 20%

**Savings rate**

\[ saverate0 \]
- **First Order:** 13%
- **Total Order:** 41%
Additional Climate Measures

Temperature 2105

CO₂ 2105

Forcing 2105

Temperature 2105

CO₂ 2105

Forcing 2105
Climate damage measures are sensitive to the same parameters over time, but the parameter interactions increase over time.
Choosing Bounds for Parameter Sampling - 1

The red lines define the bounds for the sensitivity analysis.
Choosing Bounds for Parameter Sampling - 2

Carbon Intensity of Production

Emissions from Land Use Change
Choosing Bounds for Parameter Sampling - 3

Forcing from non-CO$_2$ GHGs

Damage Factor (illustrated with deterministic case temperature increase)
Choosing Bounds for Parameter Sampling - 4

Abatement participation function

Abatement participation function showing other participation scenarios from Nordhaus (2008)
Choosing Bounds for Parameter Sampling - 5

Theta1 (cost function) for Backstop (carbon replacement)

Abatement Factor (illustrated using the cost function sensitivity bounds)
# The cost targets for the deterministic 2xCO₂ case

<table>
<thead>
<tr>
<th></th>
<th>Damages</th>
<th>Abatement</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV to 200 years</td>
<td>1.4</td>
<td>0.4</td>
<td>1.8</td>
</tr>
<tr>
<td>Decadal – 50 years</td>
<td>1.2</td>
<td>0.2</td>
<td>1.4</td>
</tr>
<tr>
<td>Decadal – 100 years</td>
<td>5.0</td>
<td>2.7</td>
<td>7.7</td>
</tr>
<tr>
<td>Decadal – 125 years</td>
<td>7.1</td>
<td>3.9</td>
<td>11.0</td>
</tr>
<tr>
<td>Decadal – 150 years</td>
<td>11.1</td>
<td>5.8</td>
<td>16.9</td>
</tr>
</tbody>
</table>

Values are trillions of 2005 US$

## Decadal costs as a % of decadal consumption:

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<tr>
<td>Decadal – 50 years</td>
<td>1.1</td>
<td>0.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Decadal – 100 years</td>
<td>2.3</td>
<td>1.3</td>
<td>3.6</td>
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<tr>
<td>Decadal – 125 years</td>
<td>2.6</td>
<td>1.4</td>
<td>4.0</td>
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<tr>
<td>Decadal – 150 years</td>
<td>2.8</td>
<td>1.5</td>
<td>4.3</td>
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